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Acoustic cyclotron resonance and giant high-frequency magnetoacoustic oscillations in metals with locally flattened Fermi surfaces

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Abstract. We consider the effect of local flattening of the Fermi surface (FS) of a metal upon geometric oscillations of the velocity and the attenuation of ultrasonic waves in the neighbourhood of the acoustic cyclotron resonance. It is shown that such peculiarities of the local geometry of the FS can lead to a significant enhancement of both the cyclotron resonance and the geometric oscillations. Characteristic features of the coupling of ultrasound to short-wavelength cyclotron waves arising due to the local flattening of the FS are analysed.

1. Introduction

The Fermi surfaces of most metals are very complex in shape and this can significantly influence observables. The phenomena which are determined by the main geometric characteristics of the FSs, i.e. their connectivity, have been well studied. However, the effects which occur due to the local geometry of the FSs such as points of flattening or parabolic points have not been investigated in detail so far. Meanwhile these local anomalies of the curvature of the FS can noticeably affect the electronic response of the metal to an external perturbation. The change in the response occurs in the nonlocal regime of propagation of the disturbance when the mean free path of electrons l is large compared to the wavelength of the disturbance λ. The reason for this is that in this nonlocal regime only those electrons whose motion is somehow consistent with the propagating perturbation can strongly absorb its energy. These 'efficient' electrons are concentrated on small 'effective' segments of the FS.

When the FS includes points of zero curvature it leads to an enhancement of the contribution from the neighbourhood of these points to the electron density of states (DOS) on the FS. Usually this enhanced contribution is small compared to the main term of the DOS which originates from all the remaining parts of the FS. Therefore it cannot produce noticeable changes in the response of the metal in the local regime of propagation of the disturbance ($l \ll \lambda$) when all segments of the FS contribute to the response functions essentially equally. However, the contribution to the DOS from the vicinities of the points of zero curvature can be congruent with the contribution of a small 'effective' segment of the FS. In other words when the curvature of the FS becomes zero at some points on an 'effective' part of the FS, it can give rise to a noticeable increase in the number of efficient electrons and, in consequence, a noticeable change in the response of the metal to the disturbance.

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The influence of locally flattened or nearly cylindrical segments of the FS on the attenuation rate and the velocity shift of ultrasonic waves propagating in a metal, as well as on its surface impedance, has been analysed before (see e.g. references [1–4]). Some results of this theoretical analysis were confirmed in experiments concerning the attenuation of ultrasonic waves in metals [5, 6]. Here we analyse the effect of the local flattening of the FS on the high-frequency magnetoacoustic oscillations.

It is known that the absorption coefficient and the velocity of sound propagating in a metal at right angles to the applied magnetic field $\boldsymbol{B}$ in the region of moderately strong magnetic fields for which the inequalities $\Omega\tau \gg 1$ and $qR \gg 1$ are satisfied simultaneously ($2R$ is the characteristic diameter of the cyclotron orbit, and q is the wave vector of the acoustic wave) oscillate as a result of variation of the magnetic field. These magnetoacoustic oscillations, which are also known as geometrical oscillations, are generated as a result of periodic reproduction of the most favourable conditions for the 'resonance' absorption of the acoustic wave energy by electrons moving along the wave front. The oscillations appear due to the commensurability of the cyclotron orbits of the electrons with the wavelength of the sound wave. Their period is determined by the extremal diameter $2R_{\mathrm{ex}}$ of the FS of the metal. The geometric oscillations exist in both low-frequency ($\omega\tau < 1$) and high-frequency ($\omega\tau > 1$) ranges (ω is the frequency of the sound wave; τ is the relaxation time). At high frequencies the magnetoacoustic oscillations may be superimposed on the acoustic cyclotron resonance. The main contribution to the oscillating corrections to the attenuation and the velocity shift originates from the vicinities of so-called stationary points of the cyclotron orbit of the extremal diameter, where an electron moves parallel to the wave front (figure 1). This leads to a conjecture that the local geometry of the FS near these stationary points will strongly affect the geometric oscillations.

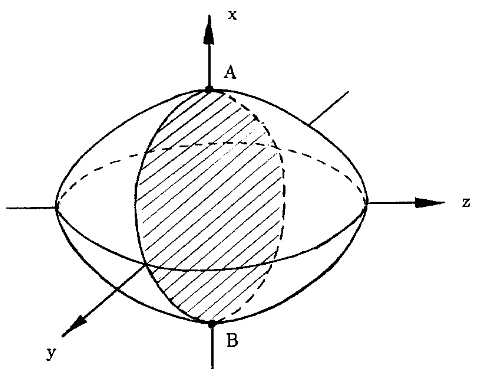

Figure 1. The axially symmetric lens corresponding to the energy–momentum relation (16). The lens is flattened at the points A $(p_2, 0, 0)$ and B $(-p_2, 0, 0)$. These points correspond to the stationary points of the cyclotron orbit of the extremal diameter when $\boldsymbol{u}_{q\omega} \parallel \boldsymbol{q} \parallel \boldsymbol{y}, \boldsymbol{B} \parallel \boldsymbol{z}$.

It follows from the theory of the geometric oscillations expounded in references [7–9] that the amplitude of oscillations in a simple metal with a closed FS has an order of magnitude smaller by a factor of $1/\sqrt{qR_{\mathrm{ex}}}$ than the smooth components of the absorption coefficient and the velocity of sound. However, in the presence of certain peculiarities in the geometry of the FS, the number of electrons effectively participating in the absorption can increase significantly, leading to an enhancement of the oscillations. It was proved, for example in [10, 11], that a sharp increase in the amplitude of geometric oscillations must take place in a conductor with a Fermi surface in the form of a slightly corrugated cylinder when the magnetic field is directed along the cylinder axis. We show below that when the FS of a metal is flattened near the points corresponding to stationary points of the cyclotron orbit of extremal diameter this also leads to a significant enhancement of the geometric oscillations.

2. Acoustoelectronic kinetic coefficients

Let us consider a longitudinal acoustic wave propagating in a metal along the y-axis of the coordinate system whose z-axis is in the direction of the magnetic field $\boldsymbol{B}$ and coincides with a high-order symmetry axis of the crystal. Assume that the elastic displacement of the lattice $\boldsymbol{u}(\boldsymbol{r}, t)$ is proportional to $\exp(\mathrm{i}qy - \mathrm{i}\omega t)$.

Proceeding from basic concepts in the theory of the propagation of ultrasound in metals [3, 12], we can write the equation for the amplitude of the elastic displacement $u_{q\omega}$ of the lattice ($u(r, t) = u_{q\omega} \exp(\mathrm{i}qy - \mathrm{i}\omega t)$) as

$$-\omega^2 \rho_m u_{q\omega} = -q^2 \rho_m s^2 u_{q\omega} + F_{q\omega} \tag{1}$$

where ρ_m is the density of matter in the lattice.

The force exerted by electrons on the lattice contains the contribution originating from their interaction with the electromagnetic field accompanying the sound wave and the deformation contribution. Correspondingly, the magnitude of this force $F_{q\omega}$ can be written as follows:

$$F_{q\omega} = \mathrm{i}q\left(\gamma_\alpha - \frac{\mathrm{i}Ne}{q}\delta_{\alpha y}\right)E'^{\alpha}_{q\omega} + \mathrm{i}\omega q^2 \beta u_{q\omega} \tag{2}$$

where

$$\boldsymbol{E}'_{q\omega} = \boldsymbol{E}_{q\omega} + \frac{\mathrm{i}\omega}{c}[\boldsymbol{u}_{q\omega} \times \boldsymbol{B}] + \frac{m}{e}\omega^2 \boldsymbol{u}_{q\omega}.$$

$\boldsymbol{E}_{q\omega}$ is the amplitude of the electric field accompanying the wave.

The amplitude $\boldsymbol{E}_{q\omega}$ satisfies the Maxwell equations

$$\left[\boldsymbol{q} \times [\boldsymbol{q} \times \boldsymbol{E}_{q\omega}]\right] = \frac{4\pi \mathrm{i}\omega \boldsymbol{J}_{q\omega}}{c^2}. \tag{3}$$

This expression contains the amplitude of the total current density $\boldsymbol{J}_{q\omega}$ induced by the passage of an acoustic wave. The components of $\boldsymbol{J}_{q\omega}$ are given by

$$J^{\alpha}_{q\omega} = \sigma_{\alpha\beta} E'^{\beta}_{q\omega} + \omega q\left(\overline{\gamma}_\alpha - \frac{\mathrm{i}Ne}{q}\delta_{\alpha y}\right)u_{q\omega}. \tag{4}$$

The electron kinetic coefficients β and σ have the form

$$\beta = \frac{\mathrm{i}}{2\pi^2\hbar^3}\int \mathrm{d}p_z\, m_\perp \sum_n \frac{U_{-n}(p_z, -q)U_n(p_z, q)}{\omega + \mathrm{i}/\tau - n\Omega} \tag{5}$$

$$\sigma_{\alpha\beta} = \frac{\mathrm{i}e^2}{2\pi^2\hbar^3}\int \mathrm{d}p_z\, m_\perp \sum_n \frac{v^{\alpha}_{-n}(p_z, -q)v^{\beta}_n(p_z, q)}{\omega + \mathrm{i}/\tau - n\Omega} \tag{6}$$

where $m_\perp$ is the cyclotron mass and $U_n(p_z, q)$ is the Fourier transform in the expansion in terms of an azimuthal angle ψ specifying the position of an electron on the cyclotron orbit:

$$U_n(p_z, q) = \frac{1}{2\pi}\int_0^{2\pi} U_n(p_z, \psi, q)\exp(\mathrm{i}n\psi)\,\mathrm{d}\psi \tag{7}$$

where

$$\begin{aligned} U_n(p_z, \psi, q) &= U(p_z, \psi)\exp\left[\mathrm{i}n\psi - \frac{\mathrm{i}q}{\Omega}\int_0^{\psi} v_y(p_z, \psi')\,\mathrm{d}\psi'\right] \\ U_n(p_z, \psi) &= \Lambda_{yy}(p_z, \psi) - \frac{\langle\Lambda_{yy}\rangle - N}{g}. \end{aligned} \tag{8}$$

Here $\Lambda_{yy}(p_z, \psi)$ and $v_y(p_z, \psi)$ are the corresponding components of the deformation potential tensor and the electron velocity, N is the electron concentration, the symbol $\langle \cdots \rangle$ denotes the averaging over the FS, g is the density of states on the FS.

The Fourier transforms of the electron velocity components in the expansion in terms of the angle ψ are determined by relations similar to (7):

$$v_n^\alpha(p_z, q) = \frac{1}{2\pi} \int_0^{2\pi} v_\alpha(p_z, \psi) \exp\left[\mathrm{i}n\psi - \frac{\mathrm{i}q}{\Omega} \int_0^{\psi} v_y(p_z, \psi')\, \mathrm{d}\psi' \right] \mathrm{d}\psi. \tag{9}$$

For a multiply connected FS, the integration with respect to p_z in (5) must be supplemented with summation over all sheets of the FS. In this case, the values of $U_n(p_z, q)$ are calculated separately for each sheet.

In equations (2), (4) the term $(\mathrm{i}Ne/q)\delta_{\alpha y}$ then has to be replaced by

$$\frac{\mathrm{i}e}{q} \sum_k N_k \frac{e_k}{|e|} \tag{10}$$

where a summation has to be performed over the sheets of the Fermi surface; N_k is the concentration of charge carriers for the kth sheet; e_k is their charge. When a metal being considered has equal numbers of electrons and holes, the term (10) is equal to zero and the corresponding addends in the expressions for $\boldsymbol{F}_{q\omega}$ and $\boldsymbol{J}_{q\omega}$ vanish. We can obtain the expression for the kinetic coefficient γ_α by replacing $U_{-n}(p_z, -q)$ by $ev_{-n}^\alpha(p_z, -q)$ in equation (5). To obtain the expression for $\overline{\gamma}_\alpha$ we have to replace $U_n(p_z, q)$ by $ev_n^\alpha(p_z, q)$.

To determine the wave vector of the acoustic wave propagating in a metal, we have to solve the equation for the amplitude of the elastic displacement of the lattice together with the Maxwell equations. As a result we arrive at the formula

$$q^2 = \frac{\omega^2}{s^2} - \frac{\mathrm{i}\omega q^2}{\rho_m s^2} \left(\beta^* + \frac{\gamma^*(\overline{\gamma}^* - Bcq/4\pi\omega)}{\sigma^* - c^2 q^2/4\pi \mathrm{i}\omega} \right). \tag{11}$$

Here

$$\begin{aligned}
\beta^* &= \beta - \left[\gamma_y - \frac{\mathrm{i}e}{q} \sum_k N_k \frac{e_k}{|e|} \right]^2 \Big/ \sigma_{yy} \\
\gamma^* &= \gamma_x - \left[\gamma_y - \frac{\mathrm{i}e}{q} \sum_k N_k \frac{e_k}{|e|} \right] \frac{\sigma_{yx}}{\sigma_{yy}} \\
\overline{\gamma}^* &= \overline{\gamma}_x - \left[\overline{\gamma}_y - \frac{\mathrm{i}e}{q} \sum_k N_k \frac{e_k}{|e|} \right] \frac{\sigma_{yx}}{\sigma_{yy}} \\
\sigma^* &= \sigma_{xx} + \sigma_{yx}^2/\sigma_{yy}.
\end{aligned} \tag{12}$$

For small amplitudes of acoustic waves, the wave vector is described by the expression

$$q = \omega/s + \Delta q. \tag{13}$$

The increment Δq linear in $u_{q\omega}$ which emerges as a result of interaction with electrons has the following form in the case under investigation:

$$\Delta q = \frac{\mathrm{i}q^2}{2\rho_m s} \left(\beta^* + \frac{\gamma^*(\overline{\gamma}^* - Bcq/4\pi\omega)}{\sigma^* - c^2 q^2/4\pi \mathrm{i}\omega} \right). \tag{14}$$

The wave vector q on the right-hand side of (14) is assumed to be equal to ω/s.

In the region under investigation where $dR \gg 1$, the main contribution to the integral with respect to ψ in expressions (7), (9) for $U_n(p_z, q)$ and $v_n^\alpha(p_z, q)$ comes from the neighbourhoods of stationary points on cyclotron orbits. Accordingly, estimating the integrals by the stationary-phase method, we can obtain the following asymptotic expressions for $U_{\pm n}(p_z, \pm q)$:

$$U_{\pm n}(p_z, \pm q) = \frac{1}{\pi} U_0(p_z) \exp\left[\pm \mathrm{i} q R(p_z) \pm \mathrm{i}\pi \frac{n}{2}\right] \\ \times \left\{\cos\left(qR(p_z) - \pi\frac{n}{2}\right) V(p_z) - \sin\left(qR(p_z) - \pi\frac{n}{2}\right) W(p_z)\right\} \tag{15}$$

where $U_0(p_z) = U(p_z, \psi_1) = U(p_z, \psi_2)$, $2R$ is the diameter of a cyclotron orbit of electrons in the direction of propagation of the acoustic wave, ψ_1 and ψ_2 are the values of the angle ψ corresponding to stationary points on the cyclotron orbit, $\psi_1 - \psi_2 = \pi$. The form of the functions $V(p_z)$ and $W(p_z)$ is determined by singularities of the energy–momentum relation for electrons in the vicinity of stationary points.

3. The model and results

Let us assume that among the sheets of a closed Fermi surface there is a biconvex lens, whose symmetry axis is the x-axis of the chosen coordinate system. We write the energy–momentum relation for the electrons associated with the lens in the form

$$E(\boldsymbol{p}) = \frac{p_1^2}{2m_1}\left(\frac{p_y^2 + p_z^2}{p_1^2}\right)^r + \frac{p_2^2}{2m_2}\left(\frac{p_x}{p_2}\right)^2. \tag{16}$$

The curvature of the FS is given by the formula

$$K = \frac{1}{v^3}(PR - Q^2) \tag{17}$$

where $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$; also,

$$\begin{aligned} P &= \left(\frac{\partial v_z}{\partial p_x} + \frac{\partial v_x}{\partial p_z}\right) v_x v_z - \frac{\partial v_z}{\partial p_z} v_x^2 - \frac{\partial v_x}{\partial p_x} v_z^2 \\ Q &= \left(\frac{\partial v_z}{\partial p_x} v_y + \frac{\partial v_z}{\partial p_y} v_x\right) v_z - \frac{\partial v_z}{\partial p_z} v_x v_y - \frac{\partial v_y}{\partial p_x} v_z^2 \\ R &= \left(\frac{\partial v_z}{\partial p_y} + \frac{\partial v_y}{\partial p_z}\right) v_y v_z - \frac{\partial v_z}{\partial p_z} v_y^2 - \frac{\partial v_y}{\partial p_y} v_z^2. \end{aligned} \tag{18}$$

Straightforward calculations give us the following result for the curvature of the lens:

$$K = \frac{r}{m_1 v^4}\left(\frac{p_y^2 + p_z^2}{p_1^2}\right)^{r-1} \left[\left(\frac{v_y^2 + v_z^2}{m_2}\right)\frac{\partial v_x}{\partial p_x} + \frac{r(2r-1)}{m_1}\left(\frac{p_y^2 + p_z^2}{p_1^2}\right)^{r-1} v_x^2\right]. \tag{19}$$

If the parameter r characterizing the shape of the lens assumes values greater than unity, then the Gaussian curvature of the surface vanishes at the points $(\pm p_2; 0; 0)$, which coincide with the vertices of the lens. Because of the axial symmetry of the lens, the vertices are points where the surface of the lens is flattened. The greater the value of r, the flatter the lens will be near its vertices. The results of experiments on the cyclotron resonance in a magnetic field applied along a normal to the surface of a metal [13, 14] give a basis for conjecture that such a flattened electron lens may be an element of the FSs of cadmium and zinc [15].

Electrons associated with the vicinities of the vertices of the lens will strongly participate in the absorption of the energy of the acoustic wave, when both the magnetic field and the acoustic wave vector are perpendicular to the axis of the lens (figure 1). Within the framework of the model (16), the functions $V(p_z)$ and $W(p_z)$ included in the expression (15) for $U_{\pm n}(p_z, \pm q)$ corresponding to the lens can be written as follows:

$$V(p_z) = \int_0^\infty \cos[qR(p_z)Q_r(y, p_z)]\,\mathrm{d}y$$
$$W(p_z) = \int_0^\infty \sin[qR(p_z)Q_r(y, p_z)]\,\mathrm{d}y \tag{20}$$

where

$$Q_r(y, p_z) = \sum_{k=1}^{r} a_k(p_z)y^{2k}\left(\frac{m_\perp^2}{m_1m_2}\right)^k \tag{21}$$

and $m_\perp$ is the cyclotron mass for the electrons associated with the lens. All of the dimensionless coefficients $a_k(p_z)$ except $a_r(p_z)$ become zero at $p_z = 0$; the latter is of the order of unity at this point. Specifically for $r = 2$ we have

$$Q_2(y, p_z) = \frac{m_\perp^2}{m_1m_2}a_1(p_z)y^2 + \left(\frac{m_\perp^2}{m_1m_2}\right)^2 a_2(p_z)y^4 \tag{22}$$

where

$$a_1(p_z) = \frac{p_z^2}{p_1^2} \qquad a_2(p_z) = \frac{1}{2}\left(1 - \frac{4}{3}\frac{p_z^4}{p_1^4}\right). \tag{23}$$

The leading term of the asymptotic expansion of the function $V(p_z)$ in inverse powers of qR originates from a neighbourhood of the central cross-section. For $r = 2$ it has the form

$$V(p_z) = \frac{\Gamma(1/4)}{4}\frac{\sqrt{m_1m_2}}{m_\perp^{\mathrm{ex}}}\left(\frac{2}{qR_{\mathrm{ex}}}\right)^{1/4}\cos\left(\frac{\pi}{8}\right). \tag{24}$$

Here, $\Gamma(x)$ is the gamma function, $m_\perp^{ex} = m_\perp(0)$, and $R_{ex} = R(0)$. The asymptotic expression for $W(p_z)$ is obtained from equation (24) by replacing the cosine by a sine of the same argument.

For an arbitrary value of r, the function $V(p_z)$ in a neighbourhood of $p_z = 0$ is described by the asymptotic expression

$$V(p_z) = \frac{1}{2r}\frac{\Gamma(1/2r)}{(qR_{ex})^{1/2r}}\sqrt{\frac{m_1m_2}{m_\perp^{ex}(a_r(0))^{1/r}}}\cos(\pi/4r). \tag{25}$$

A similar expression can also be written for $W(p_z)$.

Using the asymptotic expressions (15), (25) for $U_{\pm n}(p_z, \pm q)$ and similar asymptotics for $v_{\pm n}^{\alpha}(p_z, \pm q)$, we arrive at expressions for the electron kinetic coefficients. Taking into account that the largest contribution to the integrals over p_z in the expressions (5), (6) originates from the range of small p_z, we can replace all smooth functions of p_z in the integrands by their values at $p_z = 0$. For $qR \gg 1$ the main contribution to the asymptotic expression for β is associated with the electrons of the lens:

$$\beta = \frac{\mathrm{i}g}{\omega}\frac{\mu}{(qR_{ex})^{1/r}}U_0^2(0)X(\omega). \tag{26}$$

Here $R_{ex} = R(0)$; a dimensionless constant μ is given by

$$\mu = \frac{a_r^2}{4\sqrt{\pi}}\frac{\langle 1\rangle}{g}\Gamma\left(\frac{r+1}{2r}\right)\Big/\sqrt{\int_0^1 \overline{m}_\perp(x)\,\mathrm{d}x}. \tag{27}$$

We introduce the notation $\langle 1\rangle$ for the electron DOS on the lens; $x = p_z/p_m$; $\overline{m}_\perp(x) = m_\perp(x)/m_\perp(0)$. The frequency-dependent factor $X(\omega)$ in equation (26) has the form

$$X(\omega) = \int_{-1}^{1} Y(\omega, x)\,\mathrm{d}x \tag{28}$$

where

$$Y(\omega, x) = -\mathrm{i}\pi\frac{\omega}{\Omega}\left\{\coth\left[\pi\frac{1-\mathrm{i}\omega\tau}{\Omega\tau}\right] + \cos\left(2qR + \frac{\pi}{2r}\right)\left(\sinh\left[\pi\frac{1-\mathrm{i}\omega\tau}{\Omega\tau}\right]\right)^{-1}\right\}. \quad (29)$$

We obtain asymptotic expressions for the remaining electroacoustic kinetic coefficients in a similar way. Specifically we have

$$\gamma_y = \frac{\mathrm{i}e}{q}\sum_k N_k\frac{e_k}{|e|} + \frac{\mathrm{i}eg}{q}\frac{\mu}{(qR_{ex})^{1/r}}U_0(0)X(\omega) \quad (30)$$

$$\sigma_{yy} = -\frac{\mathrm{i}e^2}{q^2}\omega g\left(1 - \frac{\mu}{(qR_{ex})^{1/r}}X(\omega)\right). \quad (31)$$

The oscillating terms in equations (30), (31) are mainly determined by the contributions from the flattened electron lens. The contributions from remaining (nonflattened) sheets of the FS are proportional to the small factor $1/qR$ and we can omit them.

In the high-frequency range ($\omega\tau \gg 1$) the function $Y(\omega, x)$ has singularities at frequencies ω which are equal to the multiple cyclotron frequency Ω. These singularities arise due to the acoustic cyclotron resonance which was analysed in references [16, 17]. The second term in equation (29) also contains the factor $\cos(2qR + \pi/2r)$ describing geometric oscillations.

The main contribution to the integral (28) is from the region of small x where the cyclotron frequency is close to its extremum value Ω_{ex}. In this region, which corresponds to the vicinity of the central cross-section of the lens, we can use the following approximation:

$$\Omega(x) = \Omega_{ex}(1 + \eta^2 x^2) \quad (32)$$

where

$$\eta^2 = \frac{1}{\sqrt{\pi}}\frac{\Gamma((r+1)/2r)}{\Gamma(1+1/2r)}\int_0^1\frac{\mathrm{d}z}{z^2}\left(\frac{1}{\sqrt{1-z^2}} - \frac{1}{\sqrt{1-z^{2r}}}\right). \quad (33)$$

When $r = 1$ and the lens is ellipsoidal in shape, this parameter η^2 becomes zero. In this case the cyclotron frequency is independent of p_z. For a flattened lens ($r \geqslant 2$), this parameter takes nonzero values which may be of the order of unity.

3.1. Case A: moderately flattened FS

The asymptotic expression for the function $X(\omega)$ near the cyclotron resonance depends on the ratio of the parameters $2qR_{ex}$ and $(\omega\tau)^{r/2}$. Under the conditions considered both parameters are large compared to unity. Suppose that $2qR_{ex} \gg (\omega\tau)^{r/2}$. Under conditions of the acoustic cyclotron resonance in typical metals the parameter $qR_{ex} \sim v_F/s \sim 10^3$ (v_F is the Fermi velocity for the electrons associated with the lens). For $\Omega\tau \sim 10^2$ this inequality can be satisfied when the lens is moderately flattened ($1 < r < 2$). The asymptotic expression for the function $X(\omega)$ near the cyclotron resonance can be written as follows:

$$X(\omega) = \frac{\pi}{\eta}\frac{1}{\mho}\left[1 + \frac{(-1)^n b}{(qR_{ex})^{1/2l}}\frac{\cos(2qR_{ex} + \pi/4r)}{\mho}\right] \quad (34)$$

where

$$b = \frac{2\eta}{\pi}\Gamma(1 + 1/2r) \qquad \mho = \sqrt{1 - \frac{\omega}{n\Omega_{ex}} - \frac{\mathrm{i}}{\omega\tau}}.$$

The principal term in the expression obtained for $X(\omega)$ is its first term. The second term in equation (34) which describes the geometric oscillations is significantly smaller in magnitude.

When $2qR_{ex} \gg (\omega\tau)^{r/2}$ the dynamical correction Δq near the acoustic cyclotron resonance remains small compared to the main approximation for the ultrasound wave vector ω/s. For longitudinal waves this correction is mainly determined by the deformation interaction of the sound wave with the electrons. The resonance contribution to the correction Δq from the electrons associated with the neighbourhood of the central cross-section of the lens (16) equals

$$\Delta q = \gamma_0 \frac{1}{(qR_{ex})^{1/r}} \frac{qR_{ex}}{n} \frac{1}{\mho} \left[1 + \frac{b\cos(2qR_{ex} + \pi n + \pi/4r)}{\mho(qR_{ex})^{1/2r}} \right]. \tag{35}$$

Here

$$\gamma_0 = \frac{\pi N q \omega m_\perp(0)\mu}{2\eta\rho_m s^2 p_2} U_0^2(0)$$

is a quantity of the dimension and order of the attenuation rate for high-frequency ultrasound waves in the absence of the external magnetic field.

The real and imaginary parts of the correction Δq determine the resonance contributions from the electrons associated with the lens to the velocity shift $\Delta s/s$ and the attenuation rate Γ of the ultrasound wave:

$$\frac{\Delta q}{q} = \frac{\Delta s}{s} + \frac{\mathrm{i}\Gamma}{2q}. \tag{36}$$

For $r = 1$ the result (35) for the attenuation rate coincides with the corresponding result of reference [17], which is obtained under assumption that the FS of a metal has a finite and nonzero curvature everywhere. When $l = 1$ the magnitude of the resonance feature in the attenuation rate is of the order of $\gamma_0\sqrt{\omega\tau}/n$. In this case the magnitude of the geometrical oscillations is smaller by a factor $\sqrt{\omega\tau/qR_{ex}}$ than the magnitude of the resonance feature connected with the cyclotron resonance.

When $r > 1$ the effective strip on the FS passes through the flattened segments near the vertices of the lens. It gives the amplification of the acoustic cyclotron resonance. The dependence of the attenuation rate of the ultrasound on the magnetic field near the cyclotron resonance is shown in figure 2. The FS is assumed to be moderately flattened. The resonance contribution to the ultrasonic absorption coefficient increases by a factor of $(qR_{ex})^{(r-1)/r}$. This amplification arises due to the increase in the number of electrons participating in the resonance absorption of the energy of the ultrasound wave.

This increase in the number of efficient electrons also leads to amplification of the geometric oscillations. The corresponding term in equation (35) is $(qR_{ex})^{(r-1)/2}$ times larger in magnitude than a similar term in the expression for Δq for a simple metal. When the flattening of the FS becomes stronger, the magnitude of the geometric oscillations grows faster than the magnitude of the peak corresponding to the acoustic cyclotron resonance. The larger r, the larger the contribution from the term associated with the geometrical oscillations (the second term in expression (35)) to the correction Δq.

3.2. Case B: strongly flattened FS

We can use expression (35) to describe the resonance part of the dynamic correction Δq only for moderate flattening of the electron lens and moderately large $\omega\tau$. When the flattening of the lens near its vertices is strong, the quantity $(\omega\tau)^{r/2}$ exceeds the parameter $2qR_{ex}$. Under the conditions of acoustic cyclotron resonance in typical metals the inequality $2qR_{ex} \ll (\omega\tau)^{r/2}$ can be satisfied for $r > 3$.

In this case we have to use a new asymptotic expression for the function $X(\omega)$. This new asymptotic can be written as follows:

$$X(\omega) = \frac{\pi}{\eta} \frac{1}{\mho} \left[1 + (-1)^n \cos\left(2qR_{ex} + \frac{\pi}{2r} \right) \right]. \tag{37}$$

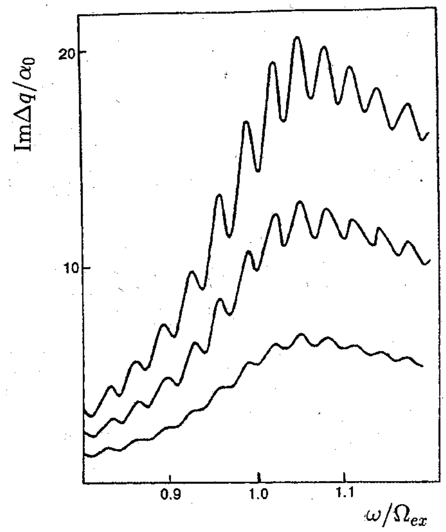

Figure 2. Attenuation of longitudinal ultrasound waves versus ω/Ω_{ex} in the vicinity of the cyclotron resonance for a moderate flattening of the Fermi surface near vertices of the electron lens. Curves are plotted for $\omega\tau = 10, qR_{ex} = 100, r = 1.25$ (curve 1), $r = 1.5$ (curve 2), $r = 1.75$ (curve 3).

The two terms in expression (37) are of the same order in magnitude. It critically changes the magnetic field dependence of the function $X(\omega)$ near the resonance ($\omega \approx n\Omega_{ex}$). When the asymptotic (37) is applicable, the factor $X(\omega)/(qR_{ex})^{1/r}$ in the expressions for the kinetic coefficients is not small compared to unity. In this connection, the contribution to the dynamic correction Δq arising due to the interaction with the electromagnetic field accompanying the sound wave becomes significant.

The effects originating from coupling of electromagnetic and ultrasound waves are well known. Specifically it has been shown that the ultrasound wave propagating perpendicularly to the external magnetic field can couple to short-wavelength cyclotron waves (see references [18–20]). In our geometry, longitudinal ultrasound waves couple to longitudinal cyclotron waves whose dispersion relation is determined by the equation $\sigma_{yy} = 0$. The dispersion curve of this mode near the frequency $n\Omega_{ex}$ can be written in the form

$$\omega_1 = n\Omega_{ex}\left[1 - \frac{1}{(qR_{ex})^{2/r}} f^2(q)\right]. \tag{38}$$

Here $f(q)$ is an oscillating function:

$$f(q) = \frac{2\pi\mu}{\eta}\cos^2\left[qR_{ex} + \frac{\pi n}{2} + \frac{\pi}{4r}\right]. \tag{39}$$

This cyclotron mode can propagate in a metal under the condition $2qR_{ex} \ll (\omega\tau)^{r/2}$. The shape of the dispersion curve of the cyclotron wave considered depends on the local geometry of the Fermi surface. Longitudinal cyclotron waves similar to the mode described by equation (38) can propagate in a metal with a spherical FS under the condition $qR_{ex} < \omega\tau$. Their dispersion relation has the form (see reference [18])

$$\omega_1 = n\Omega(1 + 1/2qR). \tag{40}$$

The differences between expressions (38) and (40) describing the dispersion curves of the longitudinal cyclotron waves are completely caused by the local flattening of the FS considered.

For a very strong flattening of the vicinities of the vertices of the electron lens ($2qR_{ex} \ll (\omega\tau)^{r/2}$) we can write the following expression for the resonance contribution to the dynamic correction Δq:

$$\Delta q = \gamma_0 \frac{qR_{ex}}{n} \frac{f^2(q)}{(qR_{ex})^{2/r}} \frac{\omega}{\omega_1 - \omega - \mathrm{i}/\tau}. \tag{41}$$

Here ω_1 is the frequency of the longitudinal cyclotron wave described by formula (41). The frequency ω_1 corresponds to the resonance rather than the cyclotron frequency Ω_{ex}. The shift of the peak of the acoustic cyclotron resonance caused by the coupling of the ultrasound to the short-wavelength cyclotron wave was studied for the spherical and ellipsoidal FSs. When the effective segments of the FS are locally flattened, this shift is more pronounced and more available for experimental observation.

Besides the cyclotron mode described by equation (41), Fermi-liquid cyclotron waves can propagate in metals. Coupling to these Fermi-liquid modes can change the resonance contribution to the dynamical correction Δq near the acoustic cyclotron resonance. However, it is shown in reference [21] that these changes are not very significant because the coupling of the ultrasound to these Fermi-liquid modes is weaker than that to the mode analysed above. This provides a reason for neglecting Fermi-liquid effects in the present consideration.

The factor $f^2(q)$ in expression (41) describes the geometric oscillations which are superimposed on the peak corresponding to the acoustic cyclotron resonance. The amplitude of these geometric oscillations sharply increases near the resonance. To order of magnitude, it is determined by the height of the resonance peak. Thus the geometric oscillations of the ultrasonic absorption coefficient in metals with strongly flattened FSs can reach values of the order of the smooth part of the attenuation rate. The geometric oscillations may become giant near the acoustic cyclotron resonance. Figure 3 illustrates this conclusion.

Since the amplification of the geometric resonances in the velocity and absorption of the ultrasound is due to the local geometric characteristics of the FS, it can be observed only for

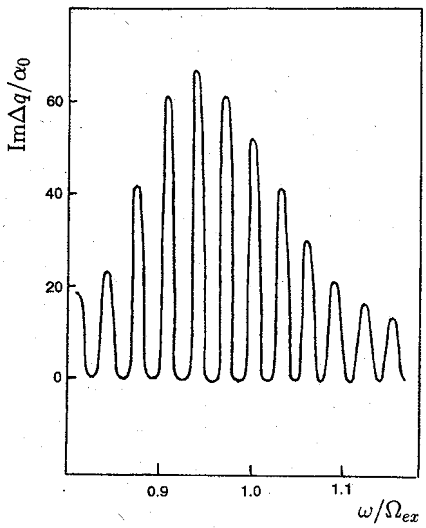

Figure 3. Giant geometrical oscillations of the attenuation rate of longitudinal ultrasound waves in the vicinity of the cyclotron resonance for strong flattening of the Fermi surface near vertices of the electron lens ($r = 4$). The curve is plotted for $\omega\tau = 10$, $qR_{ex} = 100$, $\mu = 0.1$. The shift of the resonance occurs due to the coupling of the ultrasound to the cyclotron wave.

a particular choice of the direction of the magnetic field with respect to the symmetry axes of the crystal lattice. When the magnetic field is tilted away from the direction for which the point of flattening of the FS falls on its section corresponding to the cyclotron orbit of the electrons participating effectively in the formation of the oscillations, the influence of this point vanishes and the amplitude of the oscillations decreases. Therefore, the amplification of geometric oscillations, just like a number of other effects which result from local geometric features of the FS of a metal [1–4], should exhibit a pronounced dependence on the direction of the external magnetic field. Specifically, for the model FS (16) considered here, the amplitude of the geometric oscillations of the velocity and attenuation rate of the sound will depend on the angle φ between the external magnetic field and a plane perpendicular to the axis of the lens. The range of variation of the amplitude of the oscillations on increasing φ is determined by the degree of flattening of the lens near its vertex.

4. Summary

In summary, the results of the study of electron energy spectra of metals as well as the experimental results of references [5, 6, 13, 14] provide a basis for the assumption that the FSs of certain metals (e.g. cadmium and zinc) can include locally flat segments. These features of the local geometries of the FSs can be enhanced by applying an agent that changes the shape of the constant energy surfaces, e.g., external pressure. Also, the local flattening of the FS can be enhanced in ultrathin films of metals.

We showed that the local flattening of the FS of a metal can give rise to a significant amplification of both the acoustic cyclotron resonance and the geometric oscillations of the attenuation rate and the velocity shift of the ultrasound wave. We predict that it will transpire that the amplification has to be particularly strong for a strong local flattening of the FS to occur. When the flattening of the FS at the stationary points is strong enough (within the framework of our model (16), this corresponds to $r > 3$), the magnitude of the geometric oscillations of the attenuation rate of the ultrasonic waves near the acoustic cyclotron resonance can reach values of the order of the smooth part of the absorption coefficient. We also predict that for a strongly flattened FS it will transpire that the shift in the resonance frequency of the cyclotron resonance which occurs due to the coupling of the ultrasound to the short-wavelength cyclotron wave has to be more pronounced than in typical metals and can be observed in experiments. The probability of the observation of the effect of enhancement of the geometric oscillations increases due to the anisotropy of this effect. The magnitude of the oscillations depends on the orientation of the magnetic field. The appearance of such a dependence can provide experimental evidence of the effect and also additional information on the geometry of the FS for some metals.

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